Recent applications of operations research and efficient MIP formulations in open pit mining

S. Ramazan and R. Dimitrakopoulos
Research fellow, and professor and director, respectively, W.H. Bryan Mining Geology Research Centre, The University of Queensland, Brisbane, Queensland, Australia

Abstract
Mixed integer programming (MIP) models are used for long-term production scheduling of open pit mines. However, formulations based on binary variables for mining blocks require too many variables and are often difficult or impossible to solve. This paper presents and tests alternative MIP scheduling models that have reduced the number of binary variables and solution times, thus increasing efficiency. The results serve as a guide in selecting the approach that will provide the best solution for mining any given deposit. A case study is presented to illustrate how, in the different formulations proposed, the time required for solving MIP models is reduced. In addition, proposed formulations are found to decrease the gap of feasible solutions when exact optimal solutions are difficult to obtain.

Introduction

A major element of mine planning is the optimization of long-term production scheduling. The aim is to maximize the overall discounted net revenue from a mine within operational constraints such as mining slope, grade blending, ore production and mining capacity. Mixed integer programming (MIP) and linear programming (LP) type mathematical models are considered to be powerful tools in optimizing mine scheduling, and there have been major efforts in applying them to mining projects.

Due to the difficulties inherent in solving MIP formulations containing many binary variables with the computing software and hardware available, Johnson (1968) developed an LP model for optimizing mine planning. However, this method generates partial, or fractional, mining of the blocks, causing the schedule to be sub-optimal and even infeasible when blocks are mined as a whole. Gershon (1983) discusses an MIP approach for optimizing mine scheduling that allows partial block mining on the condition that the preceding block has been fully mined. Gershon states that the model doesn't overcome the issue of solving large integer programming problems. Dagdelen (1985) applies the lagrangian method to solve an MIP model's formulations, but cannot ensure feasible solutions for all cases. In searching for alternative solutions, Dagdelen was unable to find computing capabilities to directly solve the large MIP models. Akaike (1999) developed a four-dimensional relaxation method to transform the production scheduling mathematical model into a network structure, enabling graph theory or network theory to be applied to this transformation. To reduce the effect of the gap problem, Akaike further transformed the network by considering the relaxed production capacity constraints.

These publications illustrate the importance of using MIP/LP-based mathematical modeling in mine optimization, and they note that the size of the required MIP model is a major problem because of the necessity to incorporate too many binary variables in the model. Ramazan (2001) proposes a new method based on the fundamental tree concept to decrease the number of binary variables required in MIP formulations for long-term production scheduling. Although the proposed method decreases the number of binary variables significantly, its implementation is complex. Therefore, it has not yet been applied to production scheduling in the mining industry.

This paper presents scheduling models and tests on how to generate MIP formulations using fewer binary variables. It also presents alternative approaches to MIP modeling for efficiency in solving the formulations with different mine data sets. The tests show that there are significant differences in the time taken by the various MIP models generated for the same deposit to maximize net present value (NPV). A gold mine data set is used in the case studies.

MIP formulations for long-term open pit mine production scheduling

In long-term production scheduling of open pit mines, MIP models are usually constructed to maximize the overall net present value (NPV) of the mining project. The general MIP form of open pit production scheduling is presented as follows, with some variations in the slope and reserve constraints.

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The objective function.

$$\text{Maximize} \sum_{t=1}^{p} \sum_{i=1}^{n} C_i^t * x_i^t \qquad (1)$$

where

p is the maximum number of scheduling periods;
n is the total number of blocks to be scheduled;
c_i^t is the NPV to be generated by mining block i in period t; and
x_i^t is a binary variable, which is equal to 1 if the block i is to be mined in period t and equal to 0 otherwise.

Grade blending constraints.

Upper bound constraints: The average grade of the material sent to the mill has to be less than or equal to a certain grade value, G_{max}, for each period, t

$$\sum_{i=1}^{n} \left(g_i - G_{\max}\right) * O_i * x_i^t \le 0 \qquad (2)$$

where

g_i is the average grade of block i and
O_i is the ore tonnage in block i.

Lower bound constraints: The average grade of the material sent to the mill has to be greater than or equal to a certain value, G_{min}, for each period, t

$$\sum_{i=1}^{n} \left(g_i - G_{\min}\right) * O_i * x_i^t \ge 0 \qquad (3)$$

Reserve constraints. A block cannot be mined more than once. These constraints can be formulated in two ways.

The first way is by using inequalities to state that all the blocks in the orebody model considered can be mined only once

$$\sum_{t=1}^{p} x_i^t \le 1 \qquad (4)$$

The second way is by using equalities to state that all the blocks in the model considered have to be mined once

$$\sum_{t=1}^{p} x_i^t = 1 \qquad (5)$$

Generally, the orebody model contains many blocks, and it is very difficult, or impossible, to get a solution through MIP formulations if they are applied to the whole orebody model. Thus, it is often necessary to consider applying the formulations only to the blocks within the ultimate pit limits. If the pit limits define an optimal outline, both of these reserve constraints can be considered equivalent in terms of the optimality of the solution. However, there are differences in terms of the processing (CPU) time required to solve the same model using the equality and inequality type constraints, and these are discussed in this paper.

Processing capacity constraints.

Upper bound: The total tonnage of ore processed cannot be more than the processing capacity (PC_{max}) in any period, t

$$\sum_{i=1}^{n} \left(O_i * x_i^t\right) \le PC_{\max} \qquad (6)$$

Lower bound: The total tonnage of ore processed cannot be less than a certain amount (PC_{min}) in any period, t

$$\sum_{i=1}^{n} \left(O_i * x_i^t\right) \ge PC_{\min} \qquad (7)$$

Mining capacity. The total amount of material (waste and ore) to be mined cannot be more than the total available equipment capacity (MC_{max}) for each period, t

$$\sum_{i=1}^{n} \left(O_i + W_i\right) * x_i^t \le MC_{\max} \qquad (8)$$

where

W_i is the tonnage of waste material in Block i.

To force the MIP model to produce balanced waste production throughout the periods, a lower bound (MC_{min}) may need to be implemented as follows

$$\sum_{i=1}^{n} \left(O_i + W_i\right) * x_i^t \ge MC_{\min} \qquad (9)$$

Slope constraints. All the overlying blocks that must be mined before mining a given block have to be determined. This can be implemented through one or more cone templates representing the required wall slopes of the open pit mine. There are two ways of implementing these constraints.

The first method is to use one constraint for each block per period

$$m x_k^t - \sum_{l} \sum_{r=1}^{t} x_l^r \le 0 \qquad (10)$$

where

k is the index of a block considered for excavation in Period t,
m is the total number of blocks overlying Block k and
l is the counter for the m- overlying blocks.

These types of slope constraints will be referred to as "fewer" constraints in this paper.

The second method is to use m- constraints for each block per period

$$x_k^t - \sum_{r=1}^{t} x_l^r \le 0 \qquad (11)$$

These types of slope constraints are referred to as "many" constraints in this paper.

MIP implementation with fewer binary variables

The number of binary variables required for the MIP model is equal to the number of blocks in the model multiplied by the total periods to be scheduled, as can be seen in the formulations above. For example, assume that an ultimate pit, or a pushback, containing 1,000 positive value blocks and 4,000 negative value blocks needs to be scheduled for three periods using an MIP model. To simplify the discussions, positive value blocks are referred to as "ore" and negative value blocks as "waste." Zero value blocks are considered as "air" and ignored. The traditional formulations presented suggest that the required number of binary variables for this model is 15,000.

To reduce the binary variables, the variables representing only ore blocks are defined as binary, and the remaining variables as linear. The linear variables are used to represent the percentage of the blocks to be mined in each period. The terms "partial" or "fractional" block mining are used to describe the situation where some part of a block is scheduled to be mined in one period and the rest of the block in different periods. Partial block mining is a problem if one or more of the model constraints is violated when the blocks are fully mined in one period, or if the amount of partially mined blocks is high, causing a significant violation of the optimality in a project's output.

It is interesting to analyze why solving an MIP model that has only ore blocks defined as binary can result in some of the blocks being partially mined.

One of the main reasons is that mining a block in one period is exactly the same as mining that block in any other period for the MIP's objective function. This first condition does not exist when the objective is maximizing total NPV of the project, or minimizing deviations between planned targets and actual production with different cost coefficients for different time periods. This means that mining a block in any one period is better than mining it in another period for the optimality of the objective function. Assuming the objective is maximizing the total NPV of a mine project, the MIP model will attempt to mine the blocks with positive economic values and will mine the negative value blocks only when they have to be mined due to the slope constraints. The optimality of the MIP model would result in mining the waste blocks as late as possible. Due to slope constraints and especially the time value of money, it is less costly to mine a waste block fully in the latest possible period than to mine part of it in the earlier period and part of it later for the objective function. For the optimality of the MIP model, many waste blocks will have to be fully mined. This condition is considered to be "strong" in the sense that it has strong impact in generating binary type solutions (0 – 1) for the linear variables.

The second reason for partial mining of the blocks is that many blocks have exactly the same economic value, making the model indifferent between mining one of the blocks fully and mining half from two different blocks, or mining many blocks partially adding up to the volume of one full block. This argument is acceptable on condition that there is excess mining capacity in an early period, which can be used to strip some waste blocks to enable mining of some ore blocks in future periods that could not otherwise be mined.

This second condition is considered to be "weak" in the sense that a mathematical model of a mine deposit having many waste blocks with the same economic values may still result in a binary solution or in an insignificant amount of fractional mining of waste blocks, due to the existence of slope constraints and the dominant effect of the objective function in the model, assuming that the mining capacity constraints are not very tight. If the solution of an MIP model produces partial block mining of less than an acceptable amount, which may be 5% to 10% of the waste blocks, the results can be considered satisfactory due to the fact that waste blocks do not affect the processing capacity, ore grade and quality parameters. If the results contain a high amount of fractional mining of blocks and are considered unacceptable in terms of optimality of the process, some modifications can be made to the values used in the objective function to decrease the fractional mining of waste blocks. If the objective function is NPV maximization, the economic value of the blocks can be changed by a small quantity, ε. For example, if three of the blocks have the same economic value of -10 and ε is equal to 0.001, the block values can be reset as -10.000, -9.999 and -9.998. This modification makes the block values different only for the mathematical model, but they would remain the same for the economic considerations of the mine. This minor modification of the values may significantly decrease, or sometimes avoid, the fractional mining of blocks that occurs due to blocks having the same values.

The number of binary variables required to formulate an MIP model for the given example is reduced from 15,000 to 3,000 (3*1,000) by defining only the variables of ore blocks as binary. It can also be argued that binary variables used in setting the last period's, or year's, variables can be defined as linear instead of binary. This setting did not reduce the solution time during the experiments because, if it is decided to mine a block in a given period, the variables representing all the other periods are set to zero due to equality-type reserve constraints, whether the last period is binary or not. Thus, the number of binary variables required to formulate the MIP model would only be 2,000 instead of 15,000 in the traditional approach.

In solving MIP models, the term "gap" refers to the percent difference between the best linear bound of the objective at the current stage and the current feasible binary value of the objective function (CPLEX, 1998). A gap greater than zero in a feasible binary solution of an MIP model means that the maximum possible improvement in the objective function cannot be greater than the gap. However, the objective function may or may not improve, depending on the model. In this paper, the gap is considered as an indication of the quality of a feasible solution of the MIP model.

It can be argued that it is sufficient for the mathematical model to only consider the variables of the last period in the reserve constraints if an MIP model is constructed for the blocks within an optimal pit limits. That is, the variables of the last period do not need to be considered in the objective function and the constraints related to slope, processing capacity, grade blending, mining and others. If the output of the mathematical formulation is feasible and optimal for all the periods except the last period, the blocks remaining to be mined during the last period will not change the optimality of the objective function and the slope constraints.

However, this implementation may increase the computation time required to solve the problem in some cases, because the required search area for the optimality may increase significantly, and the change in the shape of the objective function may also have negative effects. This is shown in the case study discussed below. Looking at Case 3 in Table 2, which has 0.43% gap after about 16.5 minutes of computer run, when the variables of the last period are excluded only from the objective function, it took 1.84 times longer to reach 16.7% gap and 22.5 hours to reach 12.5% gap using the same computer and software.

To determine the scheduling periods of the blocks that are scheduled for more than one period, the mathematical formulations are regenerated. In the reformulated model, the block variables that resulted in binary values in the first solution are fixed to that output value. The other block variables that resulted as partial block mining, and also the negative value

Table 1 — Gold deposit characteristics within the ultimate pit.

Characteristic	Value
Total tonnage (t)	5,710,837
Ore Tonnage (t)	3,029,156
Waste tonnage (t)	2,681,681
Average grade (g/t)	1.623
Slope in all the direction	54°
Block dimensions (m)	20 x 20 x 6
Total number of blocks	1,060
Number of ore blocks	567
Number of (+) value blocks	520
Number of waste blocks	493
Number of (-) value blocks	540
Total undiscounted value ($)	20,294,023

blocks that are not mined in the first two periods, are defined as binary variables. The second formulation is generally solved in almost no time because most of the blocks can only be assigned to one period to satisfy the model constraints. This strategy guarantees the feasibility of the output if a feasible solution exists, and it minimizes the negative impact of the partially mined blocks on the optimality of the objective.

Case study: Implementations of the MIP model

Data from a gold deposit are used to provide formulations and enable comparisons between different approaches in generating the MIP model to maximize NPV of the project. Table 1 provides information about the orebody model within the ultimate pit limits used in the case study. Grade distribution of gold is illustrated on a cross-sectional view in Fig. 1. The figure shows that part of the mine has already been mined out, exposing some of the high-grade ore material. The average grade of the ore material is around 1.6 g/t at a 1.0 g/t break-even cutoff grade, which is around 3 Mt. Total material within the final pit limits is approximately 5.7 Mt, as shown in Table 1.

This data set was chosen for testing all the different approaches used in formulating an MIP model. Most importantly, the different MIP formulations could be compared on the computation time required to solve each set of formulations and also on the gap and the number of partially mined blocks. Although the resultant NPVs from any feasible solution are very close to each other, due to the exposed high-grade ore, and they do not reflect a general realistic comparison in this specific case study, the gap and the number of partially mined blocks are good measures in general for the quality of the output.

The MIP production scheduling formulations are constructed assuming three years of mine life and with binding lower and upper constraints on the processing plant of 900,000 and 1,100,000 t/a, respectively. The long-term production scheduling process is formulated using different approaches, forming different cases as presented in Table 2. In all the cases, the MIP model contains 3,180 variables in total.

The columns in Table 2 from left to right show the case number, the number of binary variables, the total constraints, the type of reserve and slope constraints, the computation (CPU) time to solve the model, the gap, the number of partial blocks and the total NPV generated by the model. The case number x-2 refers to the reformulated model of Case x (x=1, 7, 8′ or 9′) to convert linear variables into binary. The x′ indicates that the variables representing the last period are only included in the reserve constraints for Case x, which means they are excluded from the objective function, mining capacity and the slope constraints. In the primary formulations, different numbers of binary variables refer to the different approaches used: 1,040 means the variables of positive value blocks for the first and second periods are defined as binary, 1,560 means the variables of positive value blocks for all the three periods are defined as binary, 2,120 means the variables of all the blocks for the first and second periods are defined as binary and 3,180 means all the variables are defined as binary.

In all cases, the CPLEX 6.6 program is used to solve all the MIP formulations. In solving an MIP model, if the program is not improving the solution, or is closing the gap very slowly, the program is stopped after the gap drops down to 10%. However, when the solution is too slow in improving the gap, it is stopped at the existing feasible solution status after ten hours of run, as reported in Case 10. If the program appears to be improving the gap quickly, it is stopped as soon as the gap drops below 5%. In the cases shown in Table 2, the performance is extremely slow after the reported gaps. For example, in Case 5 a solution with 0.26% gap is reached after around 172 minutes, but it took almost 27.3 hours to reach the optimal results reported in the summary of output in Table 3. The cases that took more than one hour to solve with small gaps generated the first feasible solution with less than 10% gap. The improvements in the cases that took more than one hour are pursued only if the feasible current solution had more than 10% gap. A computer with a P4 Intel processor is used in all the cases.

Table 2 — Different MIP formulations for optimization of long-term production scheduling. (See the text for explanation of the columns and symbols.)

Case	Binary variables	Total const.	Reserve const.	Slope const.	CPU time (h:m:s)	Gap, %	Partial blocks	Obj., $\$10^6$
1	1,040	3,742	=	fewer	00:02:35	1.24	19	–
1-2	408	846	=	fewer	00:00:00	0.0	0	18.28
2	1,040	23,755	=	many	01:07:39	0.15	0	18.29
3	3,180	3742	=	fewer	00:16:28	0.43	0	18.26
4	2,120	3742	=	fewer	(Same as Case 3)			
5	3,180	23,755	=	many	02:52:10	0.26	0	18.29
6	1,560	3,742	=	fewer	(Same as Case 1)			
7	1,040	3,742	≤	fewer	00:15:47	9.73	217	–
7-2	1,082	2,098	=	fewer	00:02:30	0.0	–	18.04
8′	1,040	2,852	≤	fewer	02:37:18	10.00	50	–
8′-2	578	1,188	=	fewer	00:00:00	0.0	–	18.22
9′	1,040	16,194	≤	many	00:14:07	4.56	121	–
9′-2	610	1,267	=	fewer	00:00:00	0.0	–	18.22
10	3,180	3,742	≤	fewer	16:03:44	30.14	–	17.96
11	3,180	23,755	≤	many	02:35:45	15.36	–	18.14

Comparison and analysis

In Case 1, fewer slope constraints are applied, and only the variables of positive value blocks for the first two periods are

Figure 1 — Grade distribution of gold on a south-north cross-section of the orebody model.

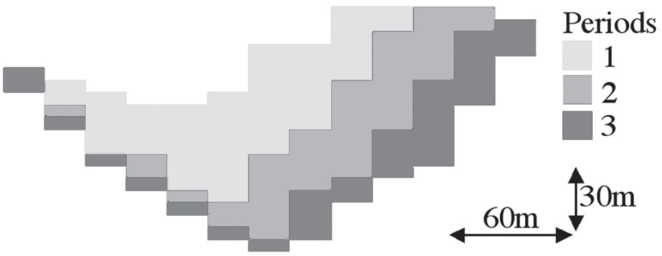

Figure 2 — South-north cross-sectional view for the optimal schedule showing the three scheduled periods of blocks

assigned as binary and the others as linear. The problem is solved in around 2.5 minutes with 1.24% gap. The solution contained 19 partially mined negative value blocks. The existence of partial block mining in this case shows that the objective function could be improved a little more if the solution could reach optimality.

Case 1-2 is generated by setting the binary variables in Case 1 to their solutions and setting partially mined blocks and also the negative value blocks that are not mined in the first two periods as binary. Case 1-2 is solved by CPLEX instantly because many of the variables can only be assigned in one period due to slope constraints. The gap and the number of partially mined blocks are so small that the results are very close to the optimum.

Case 2 is generated by using many slope constraints instead of fewer as in Case 1. In this case, the problem took more than 67 minutes to produce a solution with 0.15% gap and without any partial block mining. A similar situation appears in comparing Case 5 with Case 3, where a solution could be generated in Case 3 with fewer slope constraints about 10.4 times faster than that in Case 5 with many slope constraints. Although it was very efficient to use fewer constraints instead of many in these cases, this is not generally the rule in MIP formulations. For example, it takes more than 16 hours to generate a solution with around 30% gap using fewer slope constraints in Case 10, compared with a little more than 2.5 hours to get a solution with 15.4% gap using many constraints in Case 11. For the MIP models that are difficult to solve, or had to be run for ten hours or more, it may be better to use many slope constraints. Although this will increase the size of the model formulations, in some cases it may significantly decrease the solution time by decreasing the amount of node visiting required by the CPLEX software. Also, the many slope constraints used in Case 2 did not produce any partial block mining, which may be due to the gap being much smaller than the gap in Case 1.

Case 3 is generated by setting all the variables as binary in the MIP model. It took almost 6.4 times longer to generate a solution with 0.43% gap for Case 3 compared to Case 1. Both results are not significantly different in terms of the optimality of the output (the gap), indicating that using fewer binary variables in optimization of larger mine deposits may improve the efficiency of the solution significantly.

Case 4 is generated by setting the variables representing the last period as linear instead of binary as in Case 3. Changing the variables of the last period to linear did not improve the solution time due to the existence of reserve constraints. This means that if a solution is found for the variables of the first or second period, the variables of all other periods are directly calculated from the reserve constraints.

In a similar way, Case 6 produced the same results as Case 1. Case 6 is generated by defining the linear variables of positive value blocks in the last period of Case 1 as binary as well as the binary variables for the first and the second periods.

Case 7 is generated from Case 1 by changing the reserve constraints from equality to inequality. The program took almost six times longer than for Case 1 to find a feasible solution, which had around 9.7% gap and 217 blocks partially mined. Because inequality reserve constraints are used, the MIP model resulted in even some of the positive value blocks which are not assigned to any of the first two periods being partially mined in the last period. This case and other trials show that using equality rather than inequality reserve constraints usually brings two advantages: the program generally takes less computation time to solve the models and the models usually result in less partial block mining. In this case, the output is considered to be very poor because too many blocks are partially mined, affecting the optimality of the schedule.

Case 8′ is generated from Case 7 by excluding the variables assigned for the last period from the objective function and slope constraints. The computation time to obtain a feasible solution with 10% gap is almost 10 times more than for Case 7. Excluding some of the constraints from the model may have increased the search area causing an increase in the solution time. However, only 50 blocks are partially mined, which is much better than the 217 produced by Case 7 for a production schedule with around 10% gap. Even though there is not a significant difference between Cases 7 and 8 in terms of the gap, the quality of an output from an MIP model similar to that of Case 8 may be considered acceptable for some cases, whereas Case 7 is totally unacceptable because it has too many partially mined blocks. This shows that even a small modification in an MIP model may result in significant benefits.

Case 9′ is also generated by excluding the variables of the last period from the objective function and slope constraints and by using many slope constraints instead of fewer constraints as in Case 7. The program took a little more than 14 minutes, which is close to the 16 minutes of computation time in Case 7, to reach approximately 4.6% gap. However, the 121 partially mined blocks make the results of this case unacceptable too.

The experiments have shown that setting the variables of positive value blocks as binary and the other variables as linear may decrease the solution time significantly. Constructing the reserve constraints as equalities instead of inequalities may decrease the solution time and also play a significant role in minimizing partial mining of waste blocks. If the model is difficult to solve, as in Case 10, it may be better to use many slope constraints instead of fewer, as in Case 11.

Table 3 — Optimal scheduling output from MIP model for the gold data. The last row shows the sums for tons and economic values and average for the grade. NPV is calculated at 8% discount rate.

Periods	Ore, 10^3 t	Waste, 10^3 t	Total, 10^3 t	Grade, g/t	Value, 10^3 \$	NPV, 10^3 \$
1	1,099	583	1,682	2.002	14,643	13,558
2	1,099	1,359	2,458	1.447	4,248	3,642
3	831	739	1,570	1.355	1,402	1,113
Sum/Av	3,029	2,681	5,710	1.623	20,293	18,313

As well as helping to decrease the solution time, this may also help to decrease the partial mining of the waste blocks in some cases by generating smaller gaps. In cases where the grade blending constraints are very tight, it is suggested that all the ore blocks be defined as binary variables instead of only positive value blocks, because negative value blocks affecting the grade constraints are likely to result in partial mining.

If there are periods involving only waste stripping without producing ore in a mining operation, or if mining of waste blocks is not directly tied to the blocks with binary variables (in cases such as lateralite-type two-dimensional deposits), it may be necessary to implement the second condition, which involves changing the economic values of the waste blocks by a small amount when two or more blocks have the same value, to minimize the partial mining of waste blocks.

Table 3 shows that the ore tonnage produced as the optimal schedule during the first two years is very close to the maximum processing capacity of the processor. The total NPV value that can be generated from the schedule is about \$18.3 million, which is close to the NPVs generated by sub-optimal schedules. One of the main reasons for this is the exposing of high-grade ore that can be mined out without requiring a lot of waste stripping in this case study. Another reason is the small size of the deposit that can be mined within three years without making a big difference to the discounted economic value at 8% discount rate. The scheduling pattern in Fig. 2 shows that the exposed ore is mined within the first period, and the low-grade part of the deposit is mined in the last period.

Conclusions

Several approaches were presented for formulating an MIP model to serve as a guide for efficient optimization of long-term production scheduling in an open pit mine.

It was shown that the solution time of an MIP model can be significantly decreased by setting certain variables as binary — either only the variables of positive value blocks, or of ore blocks if the grade and/or ore production constraints are tight — and setting the other variables as linear. In most cases, the amount of partial mining of the negative value, or waste, blocks will be insignificant due to the slope constraints and the use of differentiated coefficients in the objective function between different periods.

Partial block mining can be prevented or minimized by changing the values used in the objective function by a small amount for the blocks that have the same value. This can be used if there are periods involving only waste stripping without producing ore in a mining operation, or if the total mining capacity is tight, or if mining of waste blocks is not directly tied to the blocks with binary variables (for example lateralite-type two-dimensional deposits).

It was shown that by constructing the reserve constraints as equalities instead of inequalities, the solution time can be decreased and the partial mining of waste blocks minimized. And it was shown that it may be better to use many slope constraints rather than fewer if it is difficult to solve the model because of its large size.

It should be noted that solution time in MIP models depends not only on size (number of binary variables and constraints), but also on "tightness" of the model, i.e., the data set used, the constraints and the objective function. The data used determine the shape of the objective function (coefficients in the objective function), coefficients and bounds of the constraints that have significant impact on the solution time of an MIP model. In the case studies performed, the gap in the first feasible solution was crucial in general: a small gap (less than 10%) resulted in a good solution, but a large gap (greater than 10%) could not be closed down quickly to an acceptable level. Occasionally, small changes in a mathematical model, such as the exclusion of the last period variables from one or some of the constraints and/or objective function, may result in drastic changes in the solution time and the gap on a feasible solution.

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